Flax/Epoxy Composite Beams - Simulation of Flexural Stresses

Ormondroyd, Graham; Raghavalu Thirumalai, Durai; Zhongwei, Guan

Material Today: Proceedings

DOI:
10.1016/j.matpr.2019.02.018

Published: 01/04/2019

Peer reviewed version

Flax/Epoxy Composite Beams - Simulation of Flexural Stresses

R.T. Durai Prabhakaran[a]1, Graham A Ormondroyd[a], Guan Zhongwei[b]

[a]*The BioComposites Centre, Bangor University, Gwynedd, UK*
[b]*School of Engineering, University of Liverpool, Liverpool, UK*

Abstract

Bio-based fibre reinforcements (hemp, jute, flax) derived from agricultural waste and forestry are increasing its potential towards advanced composites, promoting environmental benefits and thermal recycling (including a reduction in CO_2 emissions and the fossil fuel depletion). Currently, there is a growing interest of natural fibres due to its lower density, and higher modulus-to-density ratios compared to glass and carbon fibre reinforcements. The present work was intended to understand how the flax fibre layups and orientation affect the behaviour of laminated composites in bending. Unidirectional $[0^0]_{2S}$, cross-ply $[0/90]_S$, and angle-ply $[+45/-45]_S$ laminates made up of flax fibre reinforced epoxy composites are considered to study flexural stresses and mid-span deflections. Basic principles of the classical beam theory (CBT) are applied for obtaining analytical solutions, which were also compared with the finite element simulation results.

Keywords: Natural Fibre; Epoxy Resin; Flexural Load; Cross/Angle-ply; Classical Beam Theory; Finite Element Analysis

1. Introduction

Recent state-of-the-art reviews on bio-based composites showed a rapid growth in research and innovation in the natural fibre composite area [1-4]. Polymer and fibres derived from fossil fuel resources are mostly non-biodegradable leading to a potential increased environmental burden. Scientists from academic and industries have shifted their focus toward bio-based materials, which are more eco-friendly (low environmental impact and low cost) and hence improving commercialization of bio-based industrial products [5]. Fibres derived from agricultural waste/forestry as reinforcements have grown importance and their performance by implementing advanced chemistry and processing techniques. Henceforth bio-based composites are increasing their applications in replacing conventional composites by hybridization methods in aviation, wind, aerospace, defence and automotive areas [6].

Heterogeneous materials having two or more constituents (multi-phase) comprised of a matrix, fibre reinforcements (one or more fibres types), and nanoparticles require a thorough understanding to tailor the composite properties. Due to complex architecture, the materials have a large number of design variables. Selection of the right constituents, manufacturing methods and layups from endless combinations require modelling tools to design lightweight composites [7]. When designing such composites, the characteristics of layers should be known beforehand. Finite element method (FEM) as a numerical method offers the possibility to quickly examine and evaluate laminate design at early stages of design long before a prototype is built.

From the literature review [8-12], it is clear to note that some experimental investigations were conducted to understand stresses in a layered flax/epoxy composite. Cerbu [10] studied mechanical behaviour of flax/epoxy and flax/glass/epoxy composites, where bidirectional flax woven fabrics are considered. Performance of bidirectional eight layered flax/epoxy composites showed higher mechanical properties in weft direction when compared to warp direction. Young's modulus in tensile (33.84%)/bending (13.44%) and normal tensile stress (40.63%)/bending stress (12.69%) is greater for weft direction compared to warp direction of the specimen. Similarly, Durai Prabhakaran et. al. [11-12] studied flexural performance of biaxial ($\pm 45^0$) non-crimp glass and flax fabrics with super-sap epoxy resin. Symmetrical laminates with layered configuration are produced to demonstrate the effect of hybridization of flax/glass layups on bending properties. Limited research carried out to demonstrate the flexural performance comparison of flax/epoxy laminates with unidirectional, cross-ply, and angle-ply layups with symmetric configuration, as shown in Figure 1.

* Corresponding author. Tel.: +44-1248-382-346;
E-mail address: durai.prabhakaran@bangor.ac.uk; duraiprabhakaran@gmail.com

Theoretical and numerical analysis has been undertaken to determine stresses and mid-span deflections in layered composites in this study. Classical beam theory and finite element simulation have been used to examine the stress distribution in the symmetric laminates of flax/epoxy composites under flexural loading conditions. For the simulation of stresses, Solidworks® version 2018 software (Dassault Systems, UK) was used.

2. Symmetric Laminates and Layups

In the present paper, laminates with varying fibre orientations are considered to understand how the fibre orientation and layup sequence in a symmetric configuration affects the performance of laminated composites in bending. The laminates are shown in Figure 1, with $[0^0]_{2S}$, $[0/90]_S$, $[+45/-45]_S$ having constituent properties i.e. flax fibre and epoxy resin are described in Table 1. The four independent elastic properties for flax/epoxy lamina considered for analytical and simulation analysis are also given in Table 1.

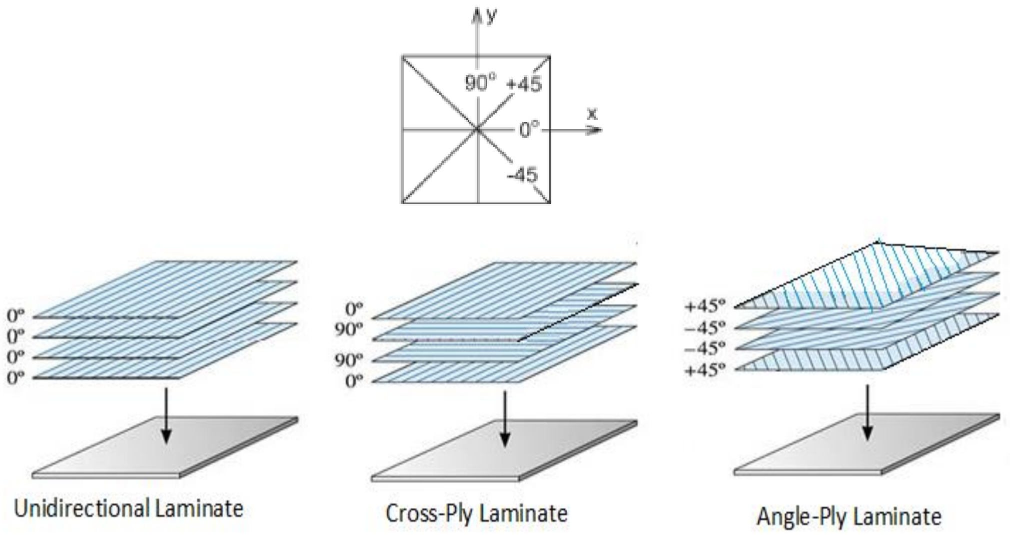


Fig. 1. Symmetric laminates: layup's with varying lamina fibre orientations.

Table 1. Material properties of fibre, matrix and lamina [11-13].

	Fibre – Flax fibre					
Symbol	V_f (%)	E_{1f} (GPa)	E_{2f} (GPa)	G_f (GPa)	ϑ_f	ρ_f (g/cm^3)
Value	0.275	39.0	5.44	3.46	0.11	1.516
	Matrix – Epoxy resin					
Symbol	V_m (%)	E_m (GPa)	-	G_m (GPa)	ϑ_m	ρ_m (g/cm^3)
Value	0.725	3.70	-	1.37	0.35	1.152
	Lamina – Flax/Epoxy single ply					
Symbol	-	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ϑ_{12}	ρ_c (g/cm^3)
Value	-	13.4	4.45	1.74	0.284	1.2794

3. Theoretical Approach

In many applications, deflection of a beam plays a key role in the structure. This can happen when the beam is subjected to either static or dynamic loading conditions. It can cause durability concerns and hence, deflection and stress analysis for composite beams need to be thoroughly understood. In the present study, composite beams with dimensions of 80mm x 15mm x 4mm are considered for three point bending analysis. Loading and supporting conditions as described in standards ISO 14125: 1998 are shown in Figure 2 used for the beam theory and finite element simulations.

According to the classical beam theory (CBT), the beam made of several layers of either same or different materials are placed either symmetrically or non-symmetrically to the median surface. In the current study, symmetric layers with the same thickness and same material are considered to define a composite laminate (Figure 2). Laminates $[0^0]_{2S}$, $[0/90]_S$, $[+45/-45]_S$ are defined as symmetric transversely orthotropic laminated conditions. The stresses in a laminate vary from layer to layer, as well strains vary linearly across the beam thickness in spite of having laminae with different directional properties [7].

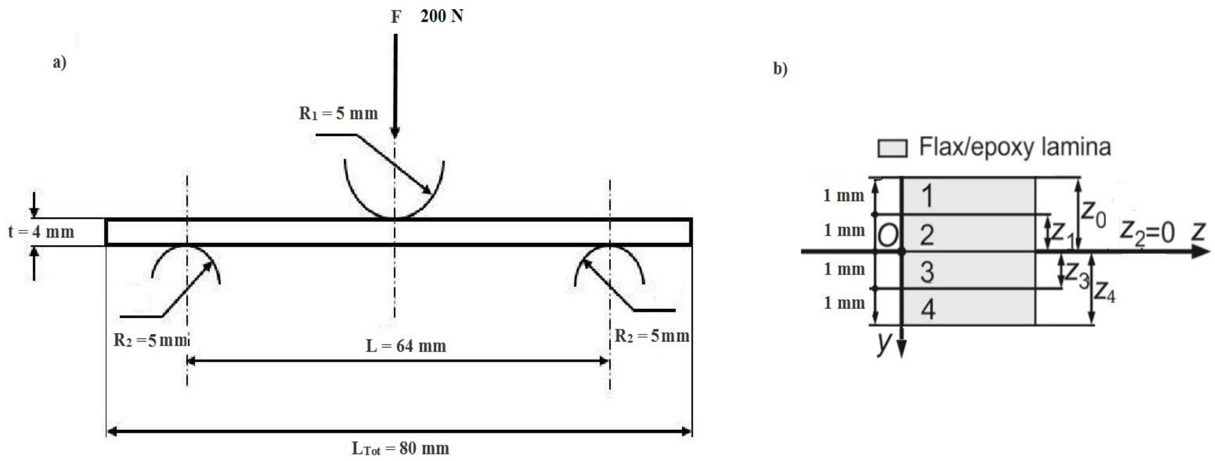


Fig. 2. a) Load configuration of 3 point bending b) Thickness and coordinates of the lamina.

From Table 1, using the constituent properties of fibre and matrix, stiffness matrix for a lamina can be defined as:

Stiffness Matrix [S] = $\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$

(1)

Where $Q_{11} = \frac{E_L}{1-\vartheta_{LT}\vartheta_{TL}}$; $Q_{22} = \frac{E_T}{1-\vartheta_{LT}\vartheta_{TL}}$; $Q_{12} = \frac{\vartheta_{TL}E_L}{1-\vartheta_{LT}\vartheta_{TL}}$; $Q_{66} = G_{LT}$;

(2)

$\vartheta_{LT}E_T = \vartheta_{TL}E_L$

(3)

According to beam theory [7, 13], total plate constitutive equation of a multi-layered laminate is used to calculate force and moment resultants

$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon_0 \\ k \end{bmatrix}$

(4)

where strains and plate curvatures are estimated by using

$\varepsilon_0 = \frac{\partial u_0}{\partial x}$ $\qquad$ $k = -\frac{\partial^2 w}{\partial x^2}$

(5)

The elements of matrix [A], [B], and [D] are defined by equation (6), as shown below :

$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k (h_k - h_{k-1})$ $\qquad$ $B_{ij} = \frac{1}{2}\sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k (h_k^2 - h_{k-1}^2)$

$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k (h_k^3 - h_{k-1}^3)$

(6)

Table 2. Transformed Stiffness Matrix of Laminates with 0, 90, 45, -45 orientations

Lamina number and fibre angle θ		$\bar{Q}_{11}$	$\bar{Q}_{22}$	$\bar{Q}_{12}$	$\bar{Q}_{66}$	$\bar{Q}_{16}$	$\bar{Q}_{26}$
		MPa					
Unidirectional Laminates $[0^0]_{2S}$							
1st, 2nd, 3rd, 4th Ply	0^0	13769	4572	1298	1740	0	0
Cross-Ply Laminate $[0/90]_S$							
1st Ply, 4th Ply	0^0	13769	4572	1298	1740	0	0
2nd Ply, 3rd Ply	90^0	4572	13769	1298	1740	0	0
Angle-Ply Laminate $[+45/-45]_S$							
1st Ply, 4th Ply	$+45^0$	6975	6975	3495	3936	2299	2299
2nd Ply, 3rd Ply	-45^0	6975	6975	3495	3936	-2299	-2299

Using equation (4) and equation (6), stress-strain relation for an orthotropic lamina referred to arbitrary axes can be determined by using equation (7)

$$\begin{Bmatrix} k_x^0 \\ k_y^0 \\ k_{xy}^0 \end{Bmatrix} = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{16} \\ \delta_{21} & \delta_{22} & \delta_{26} \\ \delta_{61} & \delta_{62} & \delta_{66} \end{bmatrix} \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$
(7)

Considering the beam section shown in Figure 2, assuming beam subjected to 3-point bending, the differential equation of deformed section of beam is derived from the classical beam theory as

$$\frac{d^2 w_0}{dx^2} = -\frac{M}{E_x I}$$
(8)

Applying simply supported beam boundary conditions and solving the above equation leads to the following **[7]**:

$$E_x = \frac{12}{h^3 \delta_{11}}$$
(9)

$$w_0 = -\frac{Fl^2}{48 E_x I} x \left[3 - \left(\frac{2x}{l}\right)^2\right] \qquad w_{0_{max}} = \frac{Fl^3}{48 E_x I}$$
(10)

$$\left(\sigma_{max}^f\right) = \frac{3F_{max} l}{2bh^2}\left(1 + 6\left(\frac{w_{max}}{l}\right)^2 - 3\left(\frac{w_{max} h}{l^2}\right)\right) \tag{11}$$

[A], [B], [D] matrices for the four layered symmetric laminates $[0^0]_{2S}$, $[0/90]_S$, $[+45/-45]_S$ can be estimated from the lamina (or each ply) transformed stiffness matrix elements as given in Table 2. Stresses in each lamina and the mid-span deflections of the composite beam are estimated for flax/epoxy symmetric laminates are given in Table 3 and Table 4. The normal stress and shear stress variations across the thickness of laminates for cross-ply and angle-ply laminates with four layers are shown in Figure 3. Ply stresses vary clearly from layer to layer as shown in Figure 3c and Figure 3d, where stresses in xy reference axes is not same as material co-ordinate systems for a layered composite.

Table 3. Stress distribution along the thickness of the laminate subjected to 3-point bending

Lamina No	Angle/fibre orientation	Stress (σ_x) MPa		Stress (σ_y) MPa		Shear Stress (τ_{xy}) MPa	
		TOP	BOTTOM	TOP	BOTTOM	TOP	BOTTOM
			Unidirectional Laminates $[0^0]_{2S}$				
Ply #1	0^0	-40	-20	0	0	0	0
Ply #2	0^0	-20	0	0	0	0	0
Ply #3	0^0	0	20	0	0	0	0
Ply #4	0^0	20	40	0	0	0	0
			Cross-Ply Laminate $[0/90]_S$				
Ply #1	0^0	-43.7	-21.9	-0.85	-0.42	-4.55e-17	-2.28e-17
Ply #2	90^0	-6.94	0	2.97	0	1.59e-16	0
Ply #3	90^0	0	6.94	0	-2.97	0	-1.59e-16
Ply #4	0^0	21.9	43.7	0.42	0.85	2.28e-17	4.55e-17
			Angle-Ply Laminate $[+45/-45]_S$				
Ply #1	$+45^0$	-38.9	-19.4	1.125	0.562	-2.57	-1.28
Ply #2	-45^0	-23.9	0	-3.94	0	8.98	0
Ply #3	-45^0	0	23.9	0	3.94	0	-8.98
Ply #4	$+45^0$	19.4	38.9	-0.562	-1.125	1.28	2.57

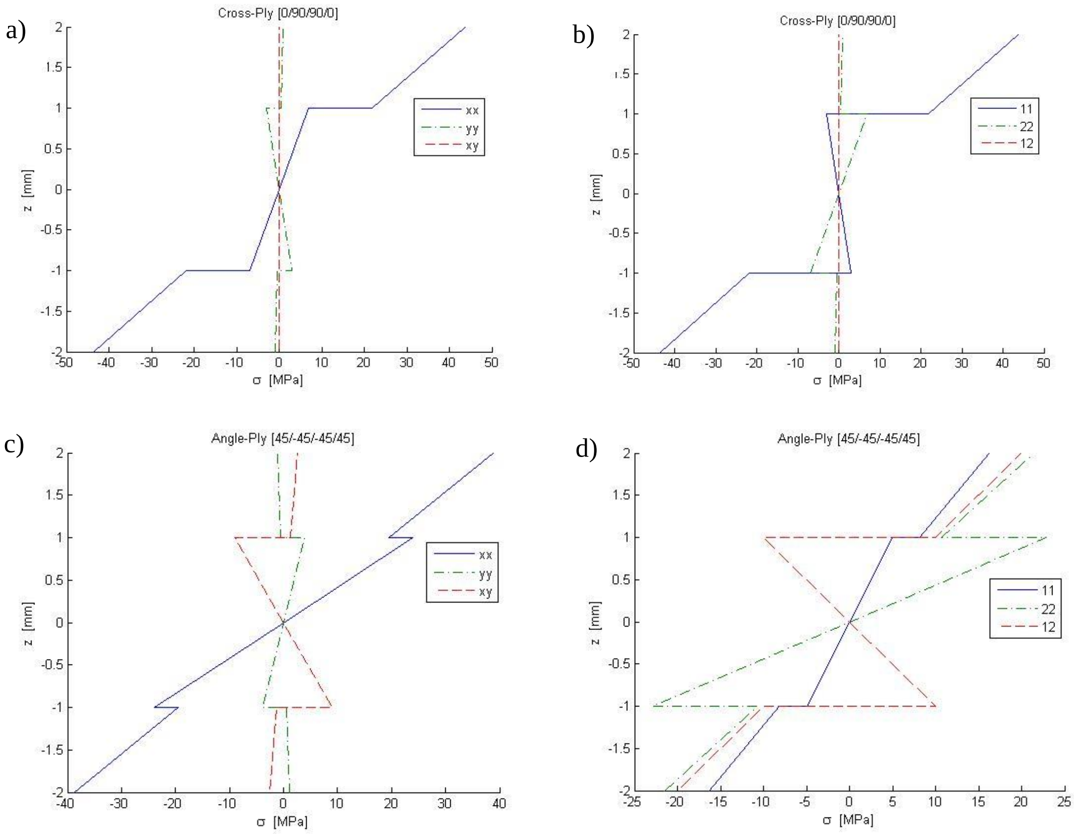


Fig. 3. The axial and shear stress variations along the thickness of cross-ply and angle-ply flax/epoxy laminate.

4. Numerical Simulation

Finite element models were developed using SOLIDWORKS® version 2018 (Dassault Systems, UK) to help understand flexural behaviour (analysis of stresses, strains and displacements) of a symmetric laminate in bending. To simulate the real material behaviour it was necessary to define density and mechanical properties of the material in Solidworks software as given in Table 1. For any simulation, geometry, material, and boundary conditions are defined. The flax/epoxy specimens considered in the study have symmetry and therefore, unidirectional and cross-ply will have symmetry in the ply sequence, material, and geometrical symmetry, whereas the angle-ply has no through thickness plane of symmetry for material orientation.

To simulate 3 point bending with the "simply supported" assumption, the load and deflection follows linear relationship. In the current study, comparison of three symmetric laminates are analysed for bending assuming load applied at centre of beam as 200N. The sum of reaction forces acting at supports are equal to 200 N in reverse direction to load applied. Specimen geometry is 80 x 15 x 4 mm3, where span (L) were set as 64 mm as shown in Figure 2. SolidWorks used the directions X, Y, and Z of the global Cartesian system of coordinates having mixed mesh (triangular shell elements) with high quality having total nodes 23208, and elements 14628.

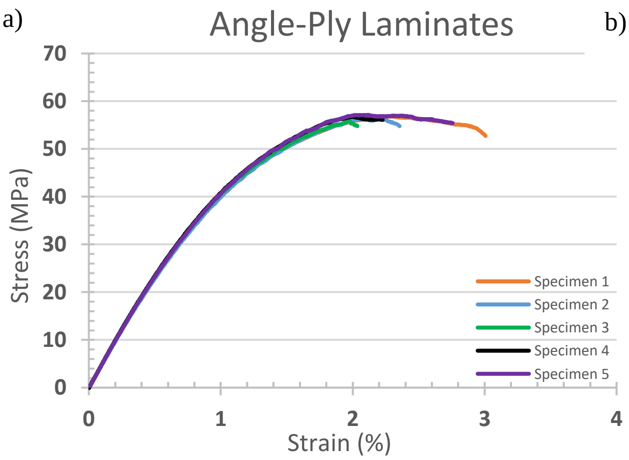
5. Experimental Work

Flax/epoxy laminates are manufacturing by vacuum assisted resin transfer molding technique. Non-crimp flax fabrics (where fibres are oriented as biaxial (±45◦)) were chosen to fabricate angle-ply laminates. Standard epoxy resin "IN2 Infusion Resin" are chosen with slow hardener, are purchased from Easy composites, UK. Slow hardener can reduce the chance of the resin 'gelling' in the pot by mixing small quantities at a time and topping up the resin pot as the resin is drawn into the laminate. The laminate thickness of 4.0mm are fabricated for 3-point bending test. Stacking sequence of the laminate is four layer layup having symmetry at mid-plane. The measured fibre volume fraction for flax fabrics is 26.7% and epoxy resin (72.8%), with a porosity of 0.5%.

Experiments are conducted to study flexural response and estimate flexural modulus and strength for the angle-ply flax/epoxy composites. Standard table-top Instron testing machine equipped with a three-point flexural testing fixture was used. Testing was performed on six specimens from the manufactured composite laminates. A crosshead speed of 2mm/min was applied. The test was conducted according to the EN ISO 14125 standard [14] using straight-sided specimens with a width-to-thickness ratio of 3. The specimen length and width were 80 and 15mm, respectively. The flexural span was 64mm. Representative measured stress-strain curves are shown in Fig. 4a. The flexural modulus was determined in the strain interval 0.0005–0.0025. In addition, the max load of the load-deflection curves was determined.

Determination of the experimental flexural modulus is based on several simplifying assumptions through the calculation of strain and stress [12]. Theoretically estimated values using CBT are consistently larger than the experimental values (in the range 5–29%), and the difference is relatively larger for composites containing flax fibre fabrics when compared with glass fabrics [12]. Figure 4b shows the flax yarns embedded in an epoxy resin have twisted fibre. Fibres having twisting in the helix shape is a major issue in composites, which result in decrease in strength and stiffness values of a composite. Resin penetration into highly twisted fibre yarns will get difficult and hence result in to poor impregnation characteristics, which can lead to porosity in the composites [13-15]. The quality of laminate fabricated by vacuum infusion is acceptable, but has 0.5% porosity content, which will decrease the mechanical properties. Fibre misalignment is another reason for decrease in mechanical performance. Even small misalignments of the fibres relative to the load direction will have a considerable influence on the mechanical properties [16]. Another major issue with natural fibres are moisture absorption. Flax and hemp yarns wet spun from boiled or bleached roving absorb more water than those obtained from raw roving, which may be attributed to the higher cellulose content of boiled and bleached yarns [17]. Moisture retaining and absorption properties can be disadvantageous to the dimensional stability of the composite as well as to the electrical resistance.

a)



b)

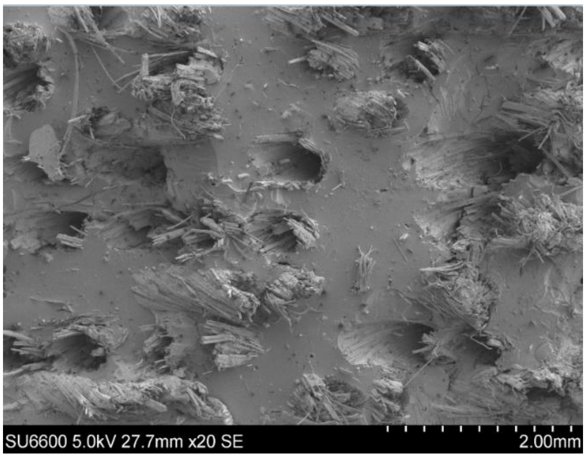


Fig. 4. a) Flexural stress-strain curves for angle-ply flax/epoxy laminates
b) Scanning electron micrograph of angle-ply flax/epoxy composite

6. Results and Discussions

The symmetry or asymmetry of a laminate based on angle, material, and thickness of plies, may cancel out some elements of the extensional stiffness [A], coupling stifness [B], and bending stiffness [D] matrices. In the current work, laminates $[0^0]_{2S}$, $[0/90]_S$, $[+45/-45]_S$ are considered as symmetry therefore elements of [B] matrix is zero and elements of Q matrix for unidirectional and cross-ply i.e. $[Q_{16}]$ and $[Q_{26}]$ are zero. According to CBT, symmetric laminates subjected to forces only have zero midplane curvatures reducing or zeroing out the coupling of forces and bending moments, normal and shear forces, or bending and twisting moments. Laminates having angle/symmetry, and number of plies the same but change the stacking sequence influences the interlaminar stresses.

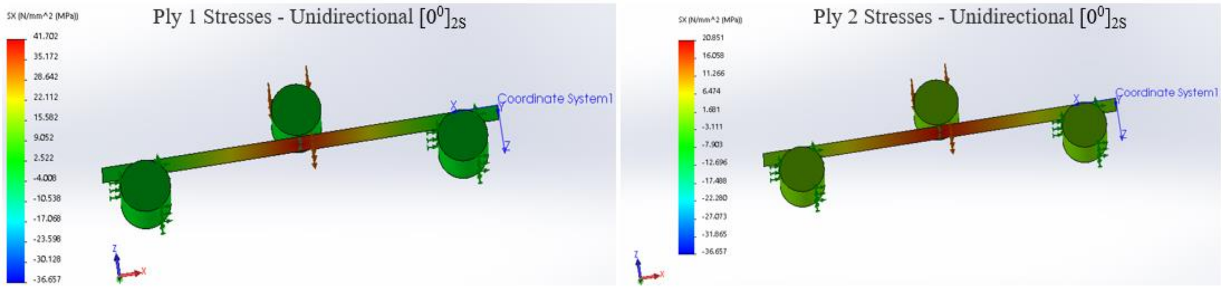


Fig. 5. Ply stresses for unidirectional $[0]_{2s}$ flax/epoxy composite.

For the current study, three point bending configuration has been adopted to study flax/epoxy laminates. To compare the laminate performance under load F (200 N) is applied at centre of beam, which is subjected to resultant moment M_x (213.3 N.m/m), M_y, M_{xy} equal to zero. As no other forces acting on beam, resultant forces N_x, N_y, N_{xy} are defined as zero, whereas the fibre direction coincides with the global axis for unidirectional and cross-ply laminates. Table 3 lists the individual ply stresses when a load is acting at centre of beam. For unidirectional and cross-ply, shear stress and midplane strains are zero because there are no in-plane forces acting and the laminate is symmetric. The state of stress through the thickness of the laminate (due to bending) results into laminate stiffness (estimated using eqn. (9). Figure 3 demonstrates the normal stress and shear stress variations across the thickness of laminates. Similarly, strain distribution through thickness of laminate is linear and plies used as outer contribute more to stiffness than inner layers of the laminate. Therefore bending stiffness for unidirectional is higher than cross-ply and angle-ply as given in Table 4. From the analytical results, it has been shown that shear stress (τ_{xy}) in angle-ply is higher than the normal stress (σ_y), and this leads to laminate twisting under bending loads. Shear stress for the unidirectional and cross-ply laminate are computed as zero, resulting no twisting.

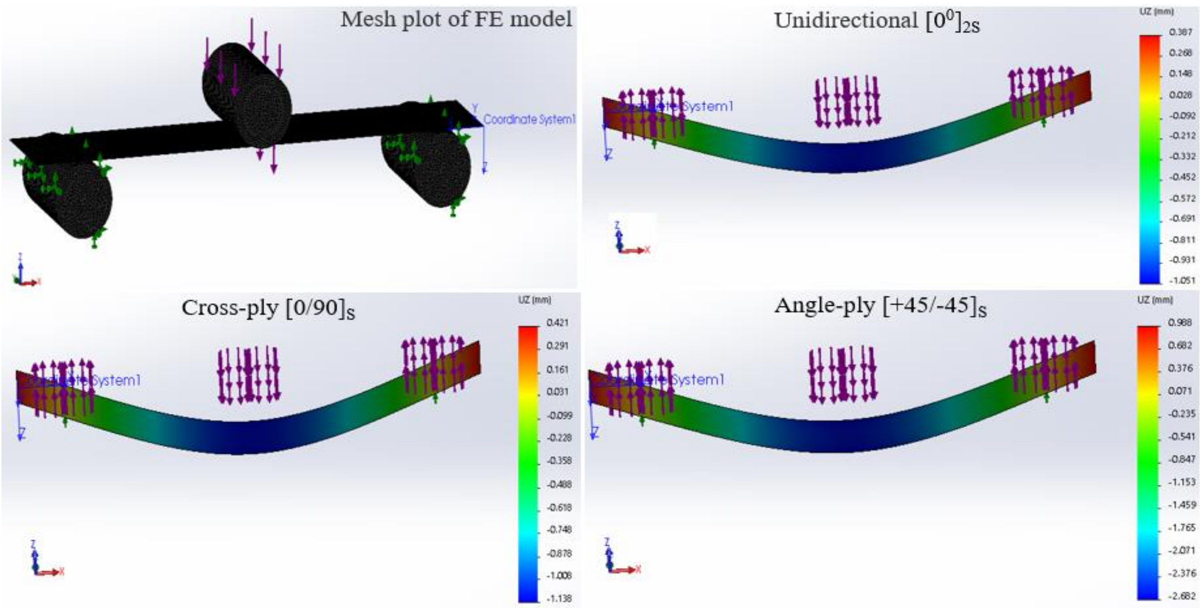


Fig. 6. Finite element model and mid-span deflection of symmetric flax/epoxy laminate.

Table 4. Bending Properties of Unidirectional, Cross-ply, & Angle-ply Flax/Epoxy Laminates.

Composite Plate Type	Bending Modulus E_x	Bending Stress σ_{fb}	Mid-Span Deflection W_{0max}		Experimental data
	Theoretical (GPa)	Theoretical (MPa)	Theoretical (mm)	Numerical (mm)	
Unidirectional $[0]_{2S}$	13.4	79.9	1.02	1.05	-
Cross Ply $[0/90]_S$	12.3	79.8	1.11	1.13	-
Angle Ply $[+45/-45]_S$	5.01	80.0	2.72	2.68	E_x = 4.0 GPa σ_{fb} = 58 MPa

Using SolidWorks, the distribution of the normal stress ($\sigma_x = S_{11}$) across the thickness of the unidirectional laminate (ply 1 and ply 2) are shown in Figure 5. The stress values shown in the plots are in the first and fourth ply as 41.7MPa and in the second and third ply as 20.8 MPa, nearly matches with the analytical (CBT) solution computed for each lamina, refer Table 3. The distribution of vertical displacement (mid-span deflection) of the laminate W_{0max} in the direction of the Z-axis are plotted in all three cases of laminate, as shown in Figure 6 and Table 4. The deflection obtained for the load (200 N) applied at centre of beam demonstrates unidirectional and cross-ply have better performance compared to angle-ply (deflection higher than $[0]_{2S}$ and $[0/90]_S$).

7. Conclusions

The work reports results obtained in numerical modelling of flexural behaviour (in bending) of flax/epoxy composites with symmetrical layups (unidirectional, cross-ply, and angle-ply) and these results are validated analytically using the classical beam theory. Based on the work, some conclusions may be drawn as follows:

- Flexural stiffness for the unidirectional laminates is greater than cross-ply and angle-ply flax/epoxy laminates.
- From the results, it has been shown that shear stress (τ_{xy}) in angle-ply is higher than the stress (σ_y), and this leads to laminate twisting under bending loads.
- The laminate (angle-ply) presents much higher mid-span deflection than symmetric unidirectional and cross-ply flax/epoxy laminates.
- Experimental results for angle-ply flax/epoxy laminates show lesser than the theoretical predictions, due to assumed perfect configurations and fibre defects exists with flax fabrics.

Future experiments are planned to study bending properties for the three symmetrical layups of flax/epoxy composites and evaluate theoretical/numerical results presented in the article.

Acknowledgements

The authors of the paper would like to acknowledge the financial support provided by the Welsh Government and Higher Education Funding Council for Wales through the Sêr Cymru National Research Network for Low Carbon, Energy and Environment.

References

[1] K. Pickering, in: Properties and performance of natural –fibre composites, 1st Edition, Woodhead Publishing, Cambridge, 2008, pp. 80 - 288.
[2] S. Edgars, Mechanical Properties of flax fibers and their composites, Licentiate thesis (Luleå University of Technology, Sweden, 2006).
[3] M. Aslan, B.F. Sørensen, and B. Madsen, Characterization of flax fibres & flax fibre composites: Bending cellulose based sources of materials,
PhD Thesis (DTU Wind Energy, Denmark, 2012).
[4] K.L. Pickering, M.G. Aruan Efendy, T.M. Le, Journal of Composites: Part A 83 (2016) 98-112.
[5] L. Yan, N. Chouw, and K. Jayaraman, Journal of Composites Part B: Engineering 56 (2014) 296 – 317.
[6] R.T. Durai Prabhakaran, International Journal of Materials Engineering Innovatio 5 (2) (2014) 81-99.
[7] B.D. Agarwal, L.J. Broutman, and K. Chandrashekhara, In: 'Analysis and performance of fiber composites', 3rd Edition, John Wiley & Sons,
Inc., New Delhi, 2006.
[8] M. Meng, H.R. Le, M.J. Rizvi, and S.M. Grove, Composite Structures 119 (2015) 693 – 708.
[9] J. George, J. Ivens, and I. Verpoest, Die Angewandte Makromolekulare Chemie 272 (1999) 41-45.

[10] C. Cerbu, Advances in Mechanical Engineering 7 (2) (2015) 01 – 11.
[11] R.T. Durai Prabhakaran, B. Madsen, H. Toftegaard, and C.M. Markussen, In proceedings of an International Conference ACMFMS 2012,
'Flexural Properties of Hybrid Natural Composite: Micromechanics and Experimental Assessment', New Delhi, December, 2012 IIT Delhi,
469 – 472.
[12] R.T. Durai Prabhakaran, H. Toftegaard, C.M. Markussen, and B. Madsen, Acta Mechanica 225 (10) (2014) 2775 – 2782.
[13] R.T. Durai Prabhakaran, M. Gupta, P. Mahajan, G. A. Ormondroyd, Intermationl Journal of Materials Engineering Innovation 2018 In Press.
[14] Fibre-reinforced plastic composites—Determination of flexural properties, International Standard ISO 14125: 1998, International Organization Standard, Geneve 20, Switzerland.
[15] M. Rask, and B. Madsen, Twisting of fibres in yarns for natural fibre composites. In: Proceedings of 18th International Conference on Composite Materials, 2011, Jeju Island, Republic of Korea.
[16] D.U. Shah, P.J. Schubel, M.J. Clifford, Journal of Composite Materials 47(4) (2013) 425–436.
[17] A. Mustata, F.St.C. Mustata, Fibres & Textiles in Eastern Europe 21, 3(99) (2013) 26-30.